

# Optimization of water use structure and plantation benefit of unit water consumption using fractional programming and conditional value-at-risk model

Fu Qiang<sup>1,2,3\*</sup>, Xiao Yuanyuan<sup>1,2,3</sup>, Cui Song<sup>1,2,3</sup>, Liu Dong<sup>1,2,3</sup>, Li Tianxiao<sup>1,2,3</sup>

(1. School of Water Conservancy and Civil Engineering, Northeast Agricultural University, Harbin 150030, China;

2. Key Laboratory of Water-Saving Agriculture of Universities in Heilongjiang Province, Northeast Agricultural University, Harbin 150030, China; 3. Key Laboratory of High Efficient Utilization of Agricultural Water Resource of Ministry of Agriculture, Northeast Agricultural University, Harbin 150030, China)

**Abstract:** For optimizing the water-use structure and increasing plantation benefit of unit water consumption, a multi-objective model for water resources utilization was established based on fractional programming (FP). Meanwhile, considering the stochasticity of water availability in the study area, the impact of the risk factor ( $\lambda$ ) from a quantitative and qualitative perspective was analyzed. The chance-constrained programming (CCP) and conditional value-at-risk (CVaR) models were introduced into five important major grain production areas in Sanjiang Plain, and the crop planting structure under this condition was optimized. The results showed that, after optimization, overall benefit of cultivation increased from 42.07 billion Yuan to 42.47 billion Yuan, water consumption decreased from 15.90 billion m<sup>3</sup> to 11.95 billion m<sup>3</sup>, the plantation benefit of unit water consumption increased from 2.65 Yuan/m<sup>3</sup> to 3.55 Yuan/m<sup>3</sup>. Furthermore, the index of water consumption, benefit of cultivation and plantation benefit of unit water consumption showed an increasing trend with the increase of violation likelihood. However, through the quantification of  $\lambda$  from an economic perspective, the increasing of  $\lambda$  could not enhance plantation benefit of unit water consumption significantly.

**Keywords:** agricultural water-use structure, plantation benefit of unit water consumption, the Sanjiang Plain, fractional programming (FP), chance-constrained programming (CCP), conditional value-at-risk (CVaR)

**DOI:** 10.3965/j.ijabe.20171002.2961

**Citation:** Fu Q, Xiao Y Y, Cui S, Liu D, Li T X. Optimization of water use structure and plantation benefit of unit water consumption using fractional programming and conditional value-at-risk model. *Int J Agric & Biol Eng*, 2017; 10(2): 151–161.

## 1 Introduction

Water is the origin of life, the key for production and the foundation of ecology. However, with industrial and agricultural development, the human population and the amount of water consumed have increased dramatically.

Limited water resources can no longer meet people's increasing needs for water. In addition, methods for water resource development in recent years have caused many ecological and environmental problems, which further exacerbate the problem of water resource scarcity, thereby severely limiting sustainable social, economic and ecological development. Furthermore, China is a

**Received date:** 2016-11-03 **Accepted date:** 2017-03-08

**Biographies:** **Xiao Yuanyuan**, Graduate student, research interests: optimization and utilization of agricultural water and soil resources and system analysis, Email: xiaoyuanyuan061@163.com;

**Cui Song**, Associate Professor, research interests: optimization and utilization of agricultural water and soil resources and system analysis, Email: cuisong-bq@neau.edu.cn; **Liu Dong**, Professor, research interests: optimization and utilization of agricultural water and soil resources and system analysis, Email: liudong5511@sina.

com; **Li Tianxiao**, Graduate student, research interests: optimization and utilization of agricultural water and soil resources and system analysis, Email: litianxiao@neau.edu.cn.

**\*Corresponding author: Fu Qiang**, PhD, Professor, research interests: optimization and utilization of agricultural water and soil resources and system analysis. Northeast Agricultural University, No.59 Mucai Road, Xiangfang District, Harbin 150030, China. Tel: +86-451-55190209, Email: fuqiang0629@126.com.

large agricultural country, and approximately 370 billion  $\text{m}^3$  of water is used for agriculture, accounting for approximately 62% of the total amount of water consumed in China. Therefore, optimizing the agricultural water-use structure and increasing plantation benefit of unit water consumption are important preconditions for achieving sustainable social development.

Whether water resources can be sustainably used directly restricts sustainable agricultural development. However, sustainable use of agricultural water resources will also affect the sustainable development of water resources, which in turn will have an impact on the sustainable development of the entire society. As the conflict between the supply and demand for water resources is becoming prominent, optimal allocation of water resources has been extensively investigated, and a number of methods for optimally allocating water resources have emerged. Lu et al.<sup>[1]</sup> established a rough-interval fuzzy linear programming model for agricultural irrigation systems. Prasad et al.<sup>[2]</sup> established a linear planning model for the optimal allocation of water resources and used this model to optimize the allocation of water at different crop growth stages under different inflow conditions. Huang et al.<sup>[3]</sup> established a model that couples two-stage stochastic programming with inexact quadratic programming and used this hybrid optimization model to predict the available amount of water that could be used for irrigation in the irrigated area in the Tarim River Basin in China. Xie et al.<sup>[4]</sup> optimized the water resources in the Nansihu Lake Basin in China using an inexact two-stage stochastic programming model. Guo et al.<sup>[5]</sup> developed an inexact fuzzy chance-constrained nonlinear programming (IFCCNP) model for agricultural water resources management under multiple uncertainties and demonstrated that the IFCCNP model was more suitable for addressing water resources allocation problems under uncertain conditions such as different precipitation and sustainable development conditions. Manoj and Mathew<sup>[6]</sup> improved water use efficiency by integrating fish culture and irrigation in coconut based farming system in Kasaragod District of Kerala (India). Cheng et al.<sup>[7]</sup> established a complex adaptive allocation model

for regional water and land resources based on the complex adaptive systems (CAS) theory and used this model to effectively mitigate the conflict between the supply of and demand for regional water resources while improving the ecological benefits. In addition, Cheng et al.<sup>[8]</sup> used the agent-based modeling method of the CAS theory to dynamically simulate four plans for using water resources and predicted the per capita food share, per capita income and water security rate in Heilongjiang Province, China, in 2020 under three climatic conditions. Fu et al.<sup>[9]</sup> optimized the allocation of an adaptive water resources management system using an interval two-stage stochastic programming model and overcame the uncertainties in the water resources system through the use of interval parameters and probability distribution. Yang et al.<sup>[10]</sup> developed an improved single-step method (SSM) based on the two-step method to solve the interval-parameter linear programming model and demonstrated that the SSM could more effectively solve irrigation water allocation optimization problems while taking into account the effect of the risk preference degree. Mirfenderesgi and Mousavi<sup>[11]</sup> developed a hybrid model by combining a particle swarm optimization algorithm with a river basin decision support system and used the hybrid model to optimize the basin-scale allocation of water resources. In addition, to achieve sustainable development, numerous researchers have started studying multi-objective programming models. Hu et al.<sup>[12]</sup> established a multi-objective programming model with economic benefits and water rights as the objective functions and used this model to balance the economic benefits and equity in water use in the Qujiang River Basin in China. Fu et al.<sup>[13]</sup> applied a cloud model to a water resource system using five sustainable water resource utilization schemes for Jiamusi, Heilongjiang Province, as an example. Li et al.<sup>[14]</sup> developed an interval linear fractional programming (FP) irrigation water allocation optimization model, and this model could quantitatively solve multi-objective problems. Ahmad and Tang<sup>[15]</sup> created a multi-objective linear programming water allocation model and introduced a compromise constraint technique and a weighting technique to optimize the allocation of water resources from the perspectives of the satisfaction level

and economic benefit. Most of the aforementioned studies only focused on the increase in the net benefit of regional water resources and neglect the increase in the net efficiency of water resources and often determined the weight based on subjective factors when handling multiple objectives. In addition, most of the aforementioned studies only sought to maximize the objective functions and neglected the water-use risk. While the FP method can effectively process uncertain information, the conventional FP method uses the expected value as the output objective, neglecting the impact of the stochastic characteristics on the output results of the model and giving no consideration to the risk resulting from the stochasticity of the data. Therefore, the conventional FP method is a risk-neutral method, although the problem of risk is the greatest concern for managers or investors.

To address the aforementioned problems, the present study selects the amount of water used for agricultural purposes and the benefit of cultivation as the objective functions and manages the coordination between these two objectives using FP, thereby avoiding the situation that occurs when using past models to solve optimization problems, in which subjective factors interfere with multiple objectives. In addition, considering the stochasticity of the amount of water that can be supplied, this study also introduces stochastic chance-constrained programming (CCP) and analyzes the optimized results of the agricultural water-use structure at different levels of the violation likelihood. Furthermore, this study also introduces a conditional value-at-risk (CVaR) model into the FP model and quantitatively analyzes the impact of the risk factor ( $\lambda$ ) on the objective functions based on the economics theory. Using the CVaR model coupled with FP, managers can not only obtain the optimal scheme but can also adjust the model by selecting a risk aversion level to obtain the optimal scheme.

## 2 Model development for optimizing agricultural water-use structure

### 2.1 FP model for optimizing agricultural water-use structure

Linear fractional programming (FP) can effectively

reflect the efficiency of a system and is advantageous in solving multi-objective programming problems, particularly agricultural water-use structure optimization problems. Therefore, this study develops an agricultural water-use structure adjustment and optimization model based on FP. This model can be calculated to obtain the minimum amount of water used, the maximum benefit of cultivation and the maximum plantation benefit of unit water consumption under the constraints of limited water resources and conditions that ensure local grain yields. The model is shown in detail as follows:

$$\max F = \frac{\max F_0}{\max F'_0} = \frac{\sum_{i=1}^I \sum_{j=1}^J \sum_{h=1}^H p_h C_{ijh} Y_{ijh} [X_{ijh}(SW) + X_{ijh}(GW)] - \sum_{i=1}^I W_i}{\sum_{i=1}^I \sum_{j=1}^J \sum_{h=1}^H p_h [X_{ijh}(SW) + X_{ijh}(GW)]} \quad (1)$$

where,  $F$  represents the objective function, Yuan/m<sup>3</sup>;  $F_0$  represents the target benefit of cultivation, 100 million Yuan;  $F'_0$  represents the target amount of water used, 100 million m<sup>3</sup>;  $p_h$  represents the probability of the occurrence of the inflow level ( $h=1, 2, 3, \dots, H$ ),  $\sum_{h=1}^3 p_h = 1$ ;  $i$  represents a certain region ( $i=1, 2, \dots, I$ );  $j$  represents the crop type ( $j=1, 2, \dots, J$ );  $Y_{ij}$  represents the unit price of the  $j^{th}$  crop in the  $i^{th}$  region, Yuan/kg;  $C_{ij}$  represents the water productivity of the  $j^{th}$  crop in the  $i^{th}$  region, kg/m<sup>3</sup>;  $X_{ij}(SW)$  represents the intermediate consumption in the  $i^{th}$  region, 100 million Yuan;  $X_{ij}(GW)$  represent the amount of surface water and groundwater that can be used for the  $j^{th}$  crop in the  $i^{th}$  region, respectively, 100 million m<sup>3</sup>.

When using a mathematical model to solve a practical problem, the parameters at the right-hand side of the constraint are oftentimes unpredictable. For example, the amount of water that can be used in a study area is stochastic and can be represented by a probability distribution. However, in many models, a fixed parameter is used to represent the amount of water that can be used to simplify the calculation process, which, to a certain degree, reduces the accuracy of the decision-making process. CCP is an effective method for solving the aforementioned problem<sup>[16,17]</sup>. Considering the stochasticity of the amount of water that

can be used, the present study introduces stochastic CCP, in which  $p_m$  is set to 0, 0.01, 0.05, 0.10, 0.15 and 0.20, respectively.  $p_m=0$  is the extreme case of CCP, the left- and right-hand sides of the constraint completely meet the requirements of the sign of inequality. The constraint conditions are reflected as follows:

1) Constraint of the amount of water supplied by the water sources:

$$\Pr \left\{ \sum_{j=1}^J X_{ijh}(SW) \leq SW_{ih} \right\} \geq 1 - p_m, \forall i, h \quad (2)$$

$$\Pr \left\{ \sum_{j=1}^J X_{ijh}(GW) \leq GW_{ih} \right\} \geq 1 - p_m, \forall i, h \quad (3)$$

$$X_{ij}(SW) + X_{ij}(GW) \leq W_{ij}, \forall i, j \quad (4)$$

where,  $m=1, 2, 3, \dots, k$  ( $k$  represents the number of constraints);  $p_m$  ( $p_m \in [0, 1]$ ) represents the preset level of probability of the  $m^{th}$  constraint; and  $SW_{ih}, GW_{ih}$  represent the total amount of surface water and groundwater that can be used in the  $i^{th}$  region at the inflow level of  $h$  respectively, 100 million  $m^3$ ;  $W_{ij}$  represents the amount of water needed for the  $j^{th}$  crop in the  $i^{th}$  region, 100 million  $m^3$ .

2) Constraint of the minimum amount of water needed:

$$X_{ij}(SW) + X_{ij}(GW) \leq W_{ijmin}, \forall i, j \quad (5)$$

3) Non-negative variable constraint:

$$X_{ij}(SW) \geq 0, \forall i, j \quad (6)$$

$$X_{ij}(GW) \geq 0, \forall i, j \quad (7)$$

where,  $W_{ijmin}$  represents the minimum amount of water needed for the cultivation of the  $j^{th}$  crop in the  $i^{th}$  region, 100 million  $m^3$ . In the model, the amount of surface water that can be supplied and the amount of groundwater that can be supplied are treated as stochastic variables. The surface water-use coefficient ( $\eta_1$ ) and the groundwater-use coefficient ( $\eta_2$ ) are 0.50 and 0.65, respectively.

## 2.2 FP model for agricultural water-use structure adjustment that considers the CVaR

The aforementioned FP model neglects the value-at-risk (VaR). Therefore, the CVaR model is

introduced to quantitatively identify the effect of  $\lambda$  on the agricultural water-use structure adjustment. The CVaR model is commonly used in financial risk management. Currently, the CVaR model is gradually being popularized in relevant fields, such as environmental science. The CVaR model was proposed based on the VaR model. The VaR model calculates the loss of the worst-case scenario that is expected to occur within the given planning period under normal conditions at a certain confidence level. Because the VaR model lacks subadditivity, the CVaR model with subadditivity is selected to evaluate the VaR of the system. The CVaR model represents the expected value of the portion of the loss that surpasses the loss determined using the VaR model. To facilitate the calculation process, the CVaR of a stochastic variable at a given confidence level  $\alpha$  ( $\alpha \in (0,1)$ ) is defined as follows:

$$\rho = \inf \left\{ \eta + \frac{1}{1-\alpha} E(P_h V_h^\pm) \right\} \quad (8)$$

where,  $\eta$  represents the maximum loss of the system at the given confidence level;  $p_h$  represents the probability of occurrence at different levels of  $h$ ;  $V_h^\pm$  represents the auxiliary variables at different levels of  $h$ <sup>[18,19]</sup>.

In addition, to investigate the response relationship between the optimal solution of the model and each parameter,  $\alpha$  is often set to 0.90, 0.95 or 0.99 based on the risk measurement model proposed by Xu and Yu<sup>[20]</sup>. Based on the actual conditions of the study area,  $\alpha$  is set to 0.95 in the present study. Furthermore,  $\lambda[\lambda \in (0,1)]$  is the risk factor that balances the benefit of cultivation and the risks. Different values of  $\lambda$  can be used as a series of risk scenarios of analysis. When  $\lambda=0$ , the maximum benefit of cultivation is generated by the water resources when the VaR is ignored. As  $\lambda$  increases, the maximum benefit of cultivation and the amount of water used calculated using the model also change accordingly. When  $\lambda=1$ , the water allocation target is close to a relatively large value. Based on the aforementioned analysis, the final expression of the model is obtained as follows:

$$\max F = \frac{\max F_0}{\min F'_0} = \frac{\sum_{i=1}^I \sum_{j=1}^J \sum_{h=1}^H p_h C_{ijh} Y_{ijh} [X_{ijh}(SW) + X_{ijh}(GW)] - \sum_{i=1}^I W_i - \lambda \left[ \eta + \frac{1}{1-\alpha} E(P_h V_h^+) \right]}{\sum_{i=1}^I \sum_{j=1}^J \sum_{h=1}^H p_h [X_{ijh}(SW) + X_{ijh}(GW)]} \quad (9)$$

### 3 Case study

#### 3.1 General information of the study area

Situated in eastern Heilongjiang Province in China (45°01'-48°27'56"N, 130°13'-135°05'26"E), the Sanjiang Plain is an alluvial plain formed by the merging and scouring of three large rivers: the Heilongjiang River, the Ussuri River and the Songhua River. The largest marshland area in China is distributed in the Sanjiang Plain. The Sanjiang Plain has a total land area of 108.90 thousand km<sup>2</sup>, 53% of which is farmland. The Sanjiang Plain is the largest food grain production base in China and can consistently provide no less than 20% of commodity grains to China each year and is the key region for achieving the objectives of the “Project of Consolidating and Improving the Hundred Billion-kg Food Grain Production Capacity” of Heilongjiang Province and the targets stipulated in the National Plan for Increasing the Food Grain Production Capacity by 100 Billion kg (2010-2020) for ensuring food security in China. Groundwater is the main source of water used for agricultural purposes in the Sanjiang Plain. More than 90% of groundwater in the Sanjiang Plain is exploited, whereas only approximately 20% of surface water is exploited and used, resulting in a relatively severe imbalance between the use of surface water and groundwater in the Sanjiang Plain.

At the same time, due to low use efficiency of surface water, and unbalanced distribution of surface water, resulting in more serious problem of water shortage in many areas. The present study selects Jiamusi, Jixi, Qitaihe, Hegang and Shuangyashan as study areas based on the acquired data and analyzes and optimizes the

agricultural water-use structure in the Sanjiang Plain. The study area and water systems are shown in Figure 1.



Figure 1 Study area and water system diagram

#### 3.2 Model parameter calibration

A frequency analysis based on historical precipitation and runoff statistical data in each administrative region in the Sanjiang Plain shows that the probability of the occurrence of medium flows is higher than that of the occurrence of high and low flows; in addition, the probability of the occurrence of high flows is essentially the same as that of the occurrence of low flows, each of which generally follow a normal distribution. Therefore, this study assumes that the probability of the occurrence of high, medium, and low inflow levels in the prediction year is 0.20, 0.60 and 0.20, respectively. In addition, because the wheat acreage in the Sanjiang Plain is less than 10% of the total acreage, the amount of water used for the cultivation of wheat is ignored. Table 1 lists the relevant data for the major crops in the study area in 2014. Table 2 lists the allowable amount of water that can be used in each administrative region at different inflow levels in the prediction year.

**Table 1 Basic data on the planting structure in the study area in Sanjiang Plain in 2014**

Administrative region	$W_i$	$C_{ij}$			$Y_{ij}$			$W_{minij}$			$A_{ij}$		
	Total cost /10 <sup>8</sup> Yuan	Unit price of crop/Yuan·kg <sup>-1</sup>			Water productivity/kg·m <sup>-3</sup>			Minimum water demand/10 <sup>8</sup> m <sup>3</sup>			Water quota/10 <sup>3</sup> m <sup>3</sup> ·hm <sup>-2</sup>		
		Rice	Maize	Soybean	Rice	Maize	Soybean	Rice	Maize	Soybean	Rice	Maize	Soybean
Qitaihe	0.6075	3.19	1.82	3.52	1.09	1.81	0.87	1.90	4.70	1.0	5.60	1.60	1.76
Jiamusi	4.0117	3.07	1.88	3.58	1.15	2.19	0.80	39.9	29.30	6.2	5.09	1.45	1.60
Shuangyashan	1.5168	3.03	1.84	3.55	1.12	2.28	0.80	8.40	14.9	1.5	5.60	1.60	1.760
Hegang	0.7527	3.06	1.87	3.52	0.88	1.53	0.60	10.20	3.00	0.4	5.09	1.45	1.60
Jixi	1.7881	3.16	1.87	3.61	1.12	2.09	1.02	21.20	11.70	2.3	5.09	1.45	1.60

Note:  $C_{ij}$  is the unit price of the  $j^{th}$  crop in the  $i^{th}$  area, Yuan/kg, the data are from the Heilongjiang Provincial Government procurement network in 2014 statistical data;  $Y_{ij}$  is the water productivity of the  $j^{th}$  crop in the  $i^{th}$  area, kg/m<sup>3</sup>, the data is comes from the 2014 Water Resources Bulletin.  $W_{minij}$  is the minimum water requirement for the  $j^{th}$  crop in the  $i^{th}$  area, which is derived from the 2014 Water Resources Bulletin.  $W_i$  is the  $i^{th}$  region of the middle consumption, 100 million Yuan, the data is taken from the survey data (taken nearly 5-year average).  $A_{ij}$  is the  $j^{th}$  crop water quota in the  $i^{th}$  area, m<sup>3</sup>/hm<sup>2</sup>, which is derived from Heilongjiang Province local water use standard DB23/T727-2009.

**Table 2 Prediction of water mining yield in the studied administrative regions under different inflow levels**

Administrative region	Inflow level	$SW_{ih}$	$GW_{ih}$	$p_{ih}$
		Available surface water supply/ $10^8 \text{ m}^3$	Available ground water supply / $10^8 \text{ m}^3$	Probability
Qitaihe	Low	9.5	4.2	0.2
	Middle	12.4	5.6	0.6
	High	14.1	6.1	0.2
Jiamusi	Low	53.1	30.2	0.2
	Middle	60.2	32.4	0.6
	High	63.7	36.6	0.2
Shuangyashan	Low	13.2	6.5	0.2
	Middle	15.5	7.3	0.6
	High	19.2	11.4	0.2
Hegang	Low	11.5	5.5	0.2
	Middle	14.4	8.3	0.6
	High	17.6	9.1	0.2
Jixi	Low	21.5	8.9	0.2
	Middle	28.9	9.7	0.6
	High	30.3	10.5	0.2

### 4 Results and discussion

A program is written using LINGO 11 to solve the agricultural water-use structure optimization model established in this study based on FP and CVaR. The optimization results are analyzed from the perspective of agricultural water-use structure, CCP and CVaR.

#### 4.1 Analysis of agricultural water-use structure

##### 4.1.1 Analysis of the amount of water used for agricultural purposes

Here, the scenario in which  $p=0$  and  $\lambda=0$  is used as an example. Table 3 lists the results of the optimization of the water supply structure for agricultural purposes in each region. As shown in Table 3, because the benefit of crops generated of unit water consumption in Qitaihe and Hegang is relatively low (Table 1), the model chooses to only meet the minimum amount of water needed ( $0.76 \text{ billion m}^3$  of water for Qitaihe and  $1.36 \text{ billion m}^3$  of water for Hegang) and does not increase the amount of water used in these two regions. In contrast, because the benefit of crops generated of unit water consumption is relatively high in Jiamusi, Shuangyashan and Jixi, the model chooses to increase the amount of water supplied at the relatively high inflow level to meet the increase in the amount of water needed for the cultivation of crops. Therefore, in balancing the comprehensive benefit of cultivation and the plantation

benefit of unit water consumption, the model chooses to increase the plantation benefit of unit water consumption instead of increasing the benefit of cultivation. The proportional structure of the amount of water supplied from surface water and groundwater in each region is plotted based on Table 3 (Figure 2). As shown in Figure 2, the percentage of the amount of surface water and groundwater used for agricultural purposes in the prediction year is 66.90% and 33.10%, respectively, at the low inflow level, 65.50% and 34.50%, respectively, at the medium inflow level, and 60.80% and 39.20%, respectively, at the high inflow level. There is a severe groundwater overexploitation problem in the Sanjiang Plain—as much as 90% of groundwater resources are being exploited in the Sanjiang Plain, which already poses a serious threat to sustainable development in the Sanjiang Plain. After optimization, the percentage of the annual amount of surface water used for agricultural purposes increases significantly, and the percentage of the annual amount of groundwater used for agricultural purposes decreases significantly at each inflow level. Therefore, these optimization results have certain practical significance for alleviating the groundwater overexploitation problem.  $SW_{ih}$ ,  $GW_{ih}$  are the total amount of surface water and groundwater when the incoming water is  $h$  in the  $i^{th}$  area, of  $100 \text{ million m}^3$ ; the data is taken from the 2014 Water Resources Bulletin.

**Table 3 Results of the planting structure optimization model based on fractional programming**

Administrative region	Crop strains	Surface water/ $10^8 \text{ m}^3$			Ground water/ $10^8 \text{ m}^3$		
		Low	Middle	High	Low	Middle	High
Qitaihe	Rice	1.013	0.875	0.846	0.887	1.025	1.054
	Maize	3.750	3.467	3.340	0.950	1.233	1.360
	Soybean	0.735	0.804	0.031	0.265	0.196	0.969
Jiamusi	Rice	18.089	22.932	15.983	6.811	1.968	8.917
	Maize	7.356	8.969	20.024	7.244	15.031	14.576
	Soybean	0.955	0.699	0.394	1.245	1.501	1.806
Shuangyashan	Rice	4.500	4.511	3.584	1.900	1.889	2.816
	Maize	8.022	10.288	14.857	3.778	4.612	7.843
	Soybean	0.679	0.701	0.759	0.821	0.799	0.741
Hegang	Rice	7.011	6.515	6.305	3.189	3.685	3.895
	Maize	1.768	1.726	1.385	1.232	1.274	1.615
	Soybean	0.168	0.216	0.203	0.232	0.184	0.197
Jixi	Rice	10.780	8.272	8.049	2.720	5.228	5.451
	Maize	2.704	7.060	7.766	1.396	1.540	2.334
	Soybean	1.016	1.269	1.284	1.284	1.031	1.016

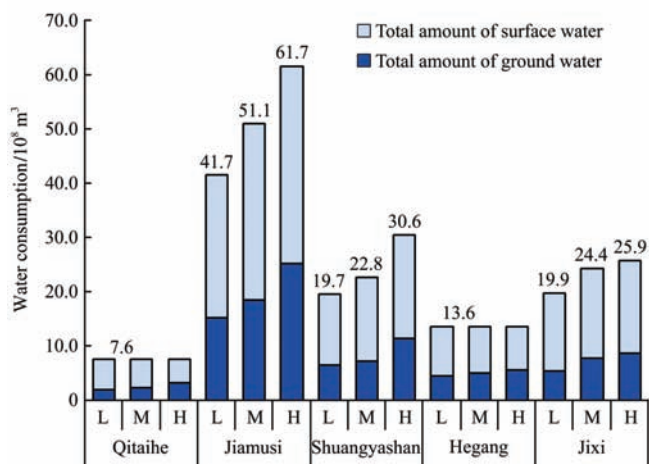


Figure 2 Optimization of water supply structures for the multi-objective model of agricultural water use based on fractional programming

4.1.2 Analysis of the cultivation scheme

To adjust the planting structure in the Sanjiang Plain by optimizing the agricultural water-use structure, the amounts of water used listed in Table 3 are converted to crop acreages based on the water quotas listed in Table 1, and the results are compared with the actual crop acreages (Table 4). As shown in Table 4, the model adjusts the overall planting structure based on the magnitude of the benefit of different crops generated of unit water consumption in different regions. The model chooses to increase the amount of water supplied for the cultivation of maize, which has a relatively high benefit generated per  $\text{m}^3$  of water. As a result, the maize acreage in Qitaihe, Jiamusi, Shuangyashan, Hegang and

Jixi increases by 9000  $\text{hm}^2$ , 208 000  $\text{hm}^2$ , 41 000  $\text{hm}^2$ , 13 000  $\text{hm}^2$  and 13 000  $\text{hm}^2$ , respectively. The model chooses to decrease the amount of water supplied for the cultivation of rice and soybean, each of which has a relatively low benefit generated of unit water consumption. Consequently, the rice and soybean acreages decrease to varying degrees. In addition, because the actual amount of water that can be supplied from surface water and groundwater is less than the minimum amount of water needed for the cultivation of crops in Jiamusi, Shuangyashan and Jixi (Tables 1 and 2), water is transferred from external water resources to meet the water requirements for the cultivation of crops in these three regions to varying degrees during the actual cultivation process (Table 5), which significantly increases the crop cultivation costs. Because the per  $\text{m}^3$  cost of transferring water from external water resources is higher than the benefit of cultivation of unit water consumption<sup>[21]</sup>, the model established in this study does not consider the transfer of water from external water resources and only uses the water resources available in each region; therefore, the optimized crop acreages in Jiamusi, Shuangyashan and Jixi are slightly less than the actual crop acreages.

**Table 4 Comparison of planting structure between optimal scheme and actual scheme in 2014**

Administrative region	Crop	Actual area / $10^8 \text{ hm}^2$	Optimal area / $10^8 \text{ hm}^2$
Qitaihe	Rice	1.8	1.4
	Maize	10.8	11.7
	Soybean	3.3	2.3
	Total area	15.9	15.4
Jiamusi	Rice	42.1	19.6
	Maize	45.2	66.0
	Soybean	17.8	5.5
	Total area	105.1	91.1
Shuangyashan	Rice	7.4	4.6
	Maize	27.6	31.7
	Soybean	4.7	3.4
	Total area	39.7	39.7

**Table 5 Diversion water usage analysis of certain administrative regions in 2014**

Administrative region	Water demand / $10^8 \text{ m}^3$	Water supply / $10^8 \text{ m}^3$	Diverted water / $10^8 \text{ m}^3$	Cost / $10^8 \text{ Yuan}$	Actual benefit / $10^8 \text{ Yuan}$
Jiamusi	77.14	51.10	26.04	104.14	173.08
Shuangyashan	23.47	22.80	0.67	2.34	85.01
Jixi	32.92	24.40	8.52	29.80	90.11



### 4.1.3 Analysis of the plantation benefit of unit water consumption

Under the constraint of limited water resources and the condition that ensures food security, the amount of water used for the cultivation of crops in each region is optimized and adjusted to obtain the maximum benefit of cultivation of unit water consumption used (Figure 3). As shown in Figure 3, the benefit of cultivation in Qitaihe, Hegang and Jixi decreases by 0.23, 0.95 and 0.20 billion Yuan, respectively, whereas the benefit of cultivation in Jiamusi and Shuangyashan increases by 1.59 and 0.20 billion Yuan, respectively; the overall benefit of cultivation increases by 0.41 billion Yuan compared with the actual benefit of cultivation (Figure 3a). In terms of the total amount of water used, the amounts of water used in Qitaihe, Jiamusi, Shuangyashan, Hegang and Jixi all exhibit a decreasing trend and decrease by 0.07, 2.60,

0.07, 0.36 and 0.85 billion m<sup>3</sup>, respectively; the total amount of water used in these regions decreases by 3.95 billion m<sup>3</sup> (Figure 3b). The plantation benefit of unit water consumption in Qitaihe decreases by 0.01 Yuan/m<sup>3</sup>, whereas the plantation benefit per m<sup>3</sup> of water in Jiamusi, Shuangyashan, Hegang and Jixi increases by 1.45, 0.19, 0.01 and 0.87 Yuan/m<sup>3</sup>, respectively; the overall benefit increases by 0.90 Yuan/m<sup>3</sup> (Figure 3c). In summary, although there is a decrease in the benefit of cultivation in Qitaihe, Hegang and Jixi and a slight decrease in the plantation benefit of unit water consumption in Qitaihe, there is an increase in the overall benefit of cultivation, a decrease in the overall amount of water used and an increase in the overall plantation benefit of unit water consumption. Therefore, when selecting between local and global optimization, the model is inclined to choose global optimization.

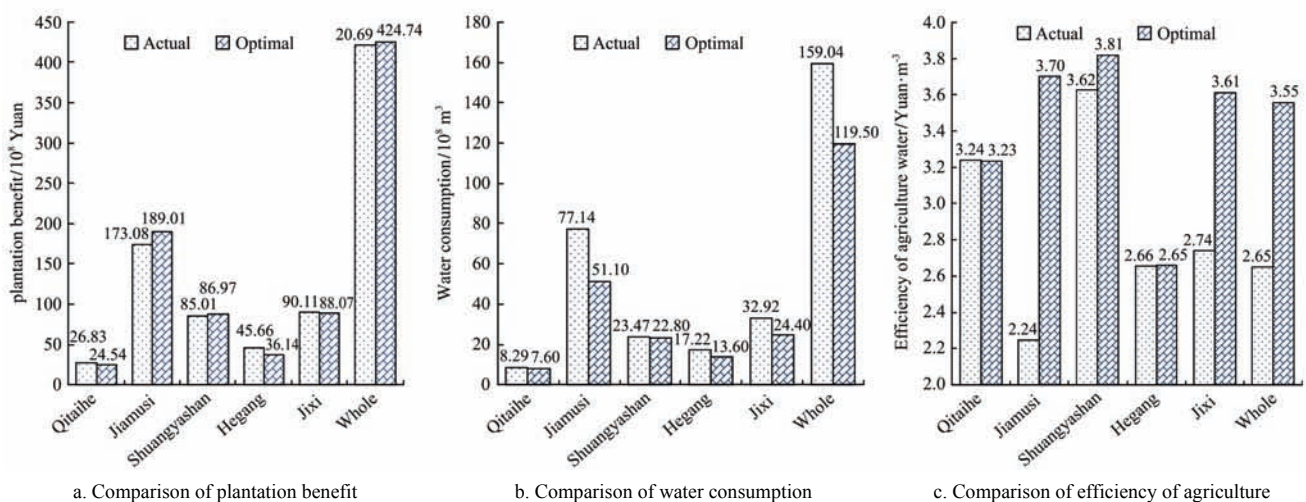


Figure 3 Comparison between the optimization of agricultural water use structure and the actual water use structure in 2014

### 4.2 Analysis of CCP

To study the effect of on the optimization results obtained from the model, CCP is introduced into the established model. The results show that the maximum benefit of cultivation, the minimum amount of water used for cultivation and the plantation benefit of unit water consumption obtained using the model change with varying levels (Figure 4). As shown in Figure 4, the overall maximum benefit of cultivation, minimum amount of water used for cultivation and plantation benefit of unit water consumption all exhibit an increasing trend with increasing and increase by 1.12 billion Yuan, 0.27 billion m<sup>3</sup> and 0.008 Yuan/m<sup>3</sup>, respectively. These changes occur because the amount of water that can be used increases with increasing, and consequently, the amount

of water that can be allocated to crops increases. In addition, because the benefit of the cultivation of maize generated of unit water consumption is greater than that of the cultivation of other crops in Shuangyashan (Table 1), as increases, the model is inclined to preferentially meet the water requirements for the cultivation of maize, which results in an increase of 0.13 billion m<sup>3</sup> in the amount of water used for the cultivation of maize, whereas the change in the amount of water used for the cultivation of other crops is relatively insignificant. However, as increases, the overall water-use risk will also increase. Therefore, decision-makers should select the balance point among the comprehensive benefit of cultivation, the plantation benefit of unit water consumption and the water-use risk when making decisions.



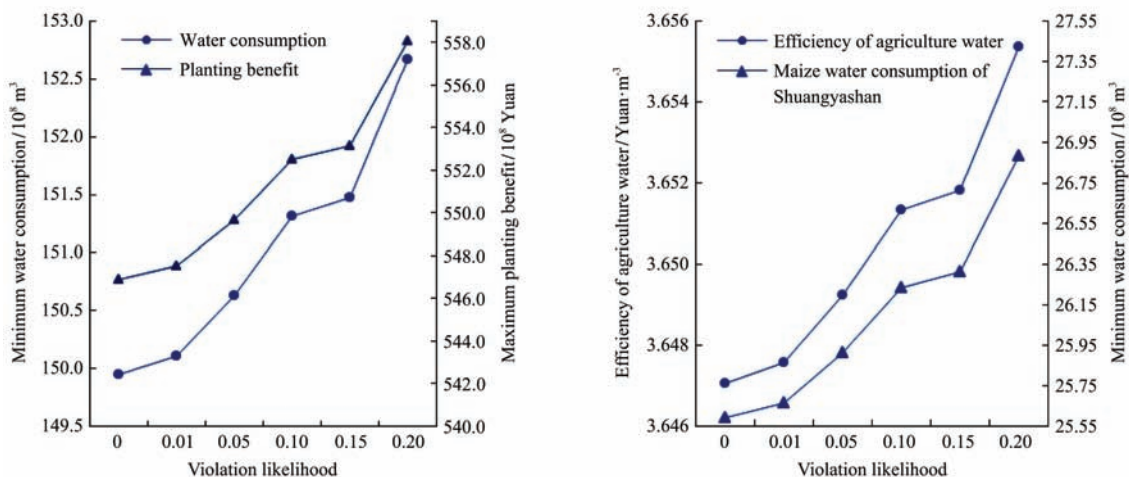


Figure 4 Maximum plantation benefit and the minimum water consumption under different violation likelihood levels

### 4.3 Analysis of the CVaR

To quantitatively calculate the impact of  $\lambda$  on the optimization results obtained from the model, the CVaR model is introduced. The following operation results are shown in Figure 5. (1) At the low inflow level, as  $\lambda$  increases, the overall VaR increases, and the increase in the maximum benefit of cultivation is less than the increase in the minimum amount of water used. As a result, the plantation benefit of unit water consumption decreases from 3.53 Yuan/m<sup>3</sup> to 3.39 Yuan/m<sup>3</sup>. (2) At the medium inflow level, when  $\lambda < 0.5$ , as  $\lambda$  increases, the overall VaR increases, and there is no significant difference between the increase in the maximum benefit of cultivation and the increase in the minimum amount of water used; in addition, there is no significant change in the overall plantation benefit of unit water consumption. When  $\lambda > 0.5$ , as  $\lambda$  increases, the VaR of the system increases, the increase in the maximum benefit of cultivation is greater than the increase in the minimum

amount of water used, and the overall plantation benefit of unit water consumption only increases by 0.04 Yuan/m<sup>3</sup>. (3) At the high inflow level, as  $\lambda$  increases, the overall VaR increases, the increase in the maximum benefit of cultivation is greater than the increase in the minimum amount of water used, and the plantation benefit of unit water consumption only increases by 0.19 Yuan/m<sup>3</sup>. Thus, a gradual increase in  $\lambda$  results in an increase in the minimum amount of water used and an increase in the maximum benefit of cultivation but does not lead to a like-for-like increase in the plantation benefit of unit water consumption, i.e., the expectation of decision-makers to increase the plantation benefit of unit water consumption cannot be achieved by increasing the amount of water used. Therefore, based on decision-makers' risk preference and different inflow levels in the prediction year, decision-makers can control the benefit of cultivation and the amount of water used but cannot substantially control the plantation benefit of unit water consumption

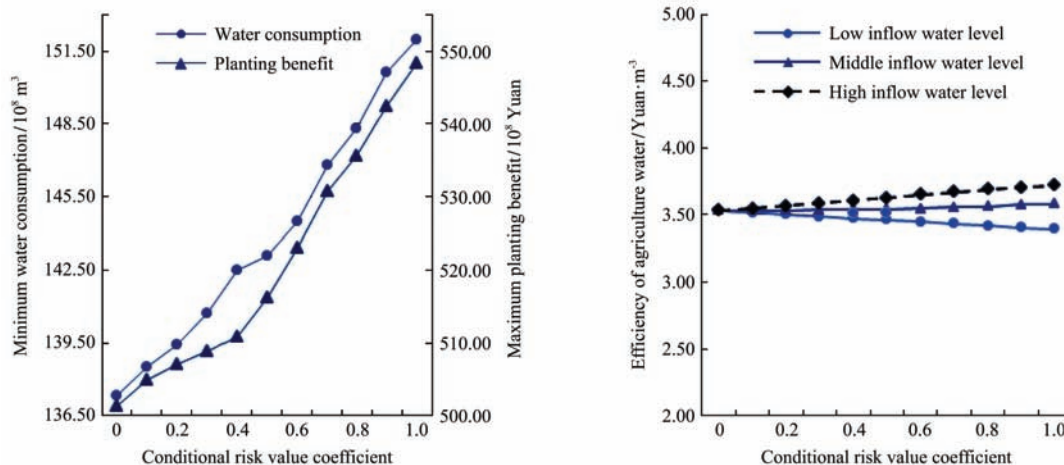


Figure 5 Efficiency of agricultural water use under different inflow water levels and different conditional risk value coefficients

## 5 Conclusions and suggestions

1) The agricultural water-use structure optimization model presented based on FP and the CVaR, which taking the benefit of cultivation as the objective in the numerator and the amount of water as the objective in the denominator of FP, thereby avoiding the subjective factors that exist in the calculation of some multi-objective models. This model can obtain the maximum benefit of cultivation and the corresponding optimal amount of water used for the cultivation of each crop under the condition of the minimum amount of water used, thereby optimizing the agricultural cultivation structure.

2) This study integrated CCP and CVaR into the established model and uses the integrated optimization model to investigate the plantation benefit of unit water consumption and obtain the pattern of the change in the plantation benefit of unit water consumption. The results demonstrate that an increase in results in an increase in the overall amount of water used, an increase in the benefit of cultivation and an increase in the overall plantation benefit of unit water consumption. However, a decrease in the risk aversion level cannot result in a significant increase in the plantation benefit of unit water consumption at different inflow levels.

3) In this study, surface water and groundwater are the only water resources considered. In reality, because some regions have relatively large acreages, the groundwater in these regions has already been severely overexploited. In recent years, these regions have started transferring water from external water resources. This study gives no consideration to the factor of water transfer from external water resources, which, to a certain extent, affects the optimization results. Therefore, it is necessary to improve the model established in the future to formulate more comprehensive schemes.

## Acknowledgments

This research has been supported by funds from National Natural Science Foundation of China (51479032, 51579044); Yangtze River Scholars in Universities of Heilongjiang Province and Water Conservancy Science

and Technology Project of Heilongjiang Province (201318, 201503); The Outstanding Youth Fund of Heilongjiang Province (JC201402).

## [References]

- [1] Lu H, Huang G, He L. An inexact rough-interval fuzzy linear programming method for generating conjunctive water-allocation strategies to agricultural irrigation systems. *Applied Mathematical Modelling*, 2011; 35(9): 4330–4340.
- [2] Prasad A S, Umamahesh N V, Viswanath G K. Optimal irrigation planning model for an existing storage based irrigation system in India. *Irrigation and Drainage Systems*, 2011; 25(1): 19–38.
- [3] Huang Y, Li Y P, Chen X, Ma Y G. Optimization of the irrigation water resources for agricultural sustainability in Tarim River Basin, China. *Agricultural Water Management*, 2012; 107: 74–85.
- [4] Xie Y L, Huang G H, Li W, Li J B, Li Y F. An inexact two-stage stochastic programming model for water resources management in Nansihu Lake Basin, China. *Journal of Environmental Management*, 2013; 127: 188–205.
- [5] Guo P, Wang X L, Zhu H, Li M. Inexact fuzzy chance-constrained nonlinear programming approach for crop water allocation under precipitation variation and sustainable development. *Journal of Water Resources Planning and Management*, 2014; 140(9): 05014003.
- [6] Samuel M P, Mathew A C. Improving water use efficiency by integrating fish culture and irrigation in coconut based farming system: A case study in Kasaragod District of Kerala (India). *Int J Agric & Biol Eng*, 2014; 7(2): 36–44.
- [7] Cheng K, Fu Q, Li T X, Jiang Q X, Liu W. Regional food security risk assessment under the coordinated development of water resources. *Natural Hazards*, 2015; 78(1): 1–17.
- [8] Cheng K, Fu Q, Chen X, Li T X, Jiang Q X, Ma X S, et al. Adaptive allocation modeling for a complex system of regional water and land resources based on information entropy and its application. *Water Resources Management*, 2015; 29(14): 4977–4993.
- [9] Qiang F, Li J, Dong L, Li T X. Allocation optimization of water resources based on uncertainty stochastic programming model considering risk value. *Transactions of the CSAE*, 2016; 32(7): 136–144. (in Chinese)
- [10] Yang G, Guo P, Li M, Fang S Q, Zhang L D. An improved solving approach for interval-parameter programming and application to an optimal allocation of irrigation water problem. *Water Resources Management*, 2016; 30(2): 701–729.
- [11] Mirfenderesgi G, Mousavi S J. Adaptive meta-modeling-based simulation optimization in basin-scale optimum water

- allocation: A comparative analysis of meta-models. *Journal of Hydroinformatics*, 2015; 18(3): 446–465.
- [12] Hu Z N, Chen Y Z, Yao L M, Wei C T, Li C Z. Optimal allocation of regional water resources: From a perspective of equity–efficiency tradeoff. *Resources Conservation & Recycling*, 2016; 109: 102–113.
- [13] Fu Q, Meng F X, Li T X, Liu D, Gong F L, Osman A, et al. Cloud model-based analysis of regional sustainable water resource utilization schemes. *Int J Agric & Biol Eng*, 2016; 9(5): 67–75.
- [14] Li M, Guo P, Singh V P. An efficient irrigation water allocation model under uncertainty. *Agricultural Systems*, 2016; 144: 46–57.
- [15] Ahmad I, Tang D. Multi-objective linear programming for optimal water allocation based on satisfaction and economic criterion. *Arabian Journal for Science and Engineering*, 2016; 41(4): 1421–1433.
- [16] Guo P, Huang G H. Two-stage fuzzy chance-constrained programming: application to water resources management under dual uncertainties. *Stochastic Environmental Research and Risk Assessment*, 2009; 23(3): 349–359.
- [17] Li Y P, Huang G H, Yang Z F, Nie S L. IFTCIP: An integrated optimization model for environmental management under uncertainty. *Environmental Modeling and Assessment*, 2009; 14(3): 315–332.
- [18] Maceira M E P, Marzano L G B, Penna D D J, Diniz A L, Justino T C. Application of CVaR risk aversion approach in the expansion and operation planning and for setting the spot price in the Brazilian hydrothermal interconnected system. *Electrical Power and Energy Systems*, 2015; 72: 126–135.
- [19] Qiu R Z, Huang X Y, Yuan H T. Robust optimization model of multi-period inventory with uncertain demand distribution. *Control and Decision*, 2014; 29(9): 1644–1648.
- [20] Xu L, Yu B. CVaR-constrained stochastic programming reformulation for stochastic nonlinear complementarity problems. *Computational Optimization and Applications*, 2014; 58(2): 483–501.
- [21] Fu Q, Zhao K, Liu D, Jiang Q X, Li T X, Zhu C H. The application of a water rights trading model based on two-stage interval-parameter stochastic programming. *Water Resources Management*, 2016; 30(7): 2227–2243.